17 Investment planning in market socialism

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1 Introduction

Despite the fact that the phrase "market socialism" has a respectable history, dating back to Lange's (1938) famous contribution in the Austrian debate, there has been little formal economic theorizing on the nature of market socialism.[1] We take market socialism to be a system of economic organization in which (1) the state has the authority to influence the pattern and levels of investment across sectors; (2a) most, if not all, resources are distributed via markets; (2b) citizens, in particular, earn income from labor that is traded on markets; (3a) firms are publicly owned, which means that profits are distributed to members of the population in proportions that are politically determined; and (3b) firms maximize profits. In this chapter, we depart from much contemporary literature (e.g., Miller, 1990 and Estrin and LeGrand, 1989, but not Brus and Laski, 1990), and do not consider the presence of worker-managed firms to be a necessary condition of market socialism.

Given the above characterization of market socialism, two kinds of question naturally arise. The first concerns the *consistency* of the five characteristics listed above. For example, can one design institutions which will assure that firms maximize profits while at the same time distributing profits diffusely among the population rather than concentrating firm ownership in the hands of stockholders who can trade stock? Is it possible for the government to control the distribution of corporate profits, without interfering with profit-maximization? It is largely these questions that have animated the debate on market socialism in recent years. The second kind of question concerns the *flexibility* of market socialism. Assuming that the incentive problems raised by the first kind of question can be solved, to what extent can a market socialist economy differ from a capitalist economy? In particular, what scope is there for planning in an economy where most private goods are allocated through markets? If the answer to the second question were "very little," then there would be little reason to advocate a market socialist economy. Social democracy, a system with significant redistribution of income but in which investment decisions are still made by capitalists, would be no different from market socialism, and probably easier to implement.

Our focus in this chapter is on the second kind of question. Our modeling choice has been to treat the structure of investment as the planning objective, although we could have chosen other objectives instead, such as the level of employment or the structure of employment across industries. Our treatment of the problem is classical: we attempt to characterize the class of investment vectors that can be implemented in a market economy if the government has available various types of price and quantity instruments. We have chosen investment as the focus, for we believe it is arguably the aspect of capitalist economies that most justifies intervention. In large economies, at least, investment leads the business cycle. Furthermore, there are significant externalities, both positive and negative, associated with investment that may justify government intervention.[2] We do not debate here with those who argue that government intervention in the investment process can only lower social welfare from what it would be without intervention.

Let there be an investment-good industry, sector 0, and N sectors which use the investment good as an input and produce consumer goods. The economy exists for two periods; in the first period, firms place orders for the investment good which will increase their productivity in the second period. In a private-ownership model of the economy, there will be a Walrasian equilibrium in which a certain vector of investments $\bar{I}^W = (\bar{I}_1^W, \ldots, \bar{I}_N^W)$ is attained in the consumer-good industries. The socialist state, however, wishes to implement a different vector of investments, say $\bar{I}$.

How can the center implement the sectoral investment $\bar{I}$? We assume that it has available various price or quantity *instruments* and various options for *taxation*; as summarized in Figure 17.1, we characterize the class of implementable investment vectors for three combinations of these instruments, indicated by the filled-in cells of Figure 17.1.

We view Lange's (1938) *On the Economic Theory of Socialism* as the direct ancestor of our own study. Despite the claimed parentage, Lange's concerns were quite different from ours. He facilely remarked that the center could set the interest rate to implement the level of total investment that it wants to attain in the economy – that was the extent of his concern with the implementation problem, as we have defined it. In section 4 we present a model which generalizes Lange's suggestion. The set of instruments consists of N discounts on the market interest rate – one for the firm in each consumer-good sector. Taxes are levied on firm profits to balance the deficit of the central bank. Firms borrow to finance investment; they maximize profits in the presence of markets for all commodities. Each citizen receives, as well as wage income, a fraction of total after-tax profits, what Lange called the social dividend, which is determined by some political process.

Lange did not study the set of implementable investment vectors, but was concerned with the calculation of prices. He remarked that the prices of consumer goods could well be set by markets, but the prices of investment goods were too important to be left to the market, and should be set by the center *equal to their values at market equilibrium*. Evidently, he believed that disequilibrium in the investment-goods' sectors was too destabilizing for the economy, and that a considerable advantage of socialism over capitalism would be the setting of these prices by the center instead of the market. Thus, he proposed a tâtonnement mechanism by which the state could calculate prices that would converge, he

	instruments		
	prices		quantities
	consumption goods	investment goods	
corporation tax		Lange model (i)	direct investment model (i)
sales tax	sales-tax model		

Figure 17.1 The class of implementable investment vectors

assumed, to equilibrium.[3] In the literature that followed Lange (notably, Arrow and Hurwicz, 1958, and Arrow, Block and Hurwicz, 1959), the question of the convergence of tâtonnement was studied rigorously. But the basic issue of implementation has never been raised in the literature on market socialism.

Economists have since become less interested in tâtonnement, because of the results of Debreu (1974) and Sonnenschein (1973a, 1973b) demonstrating that on the space of economic environments, the convergence of the tâtonnement process to Walrasian equilibrium is the exception rather than the rule. In our chapter, we study only the existence of equilibrium. We think of the center as setting the values of its instruments, and then view the existence of an equilibrium at those parameter values as a stylization of the conventional wisdom that "the market finds equilibrium."[4]

There is an important inconsistency between the motivation of our study and the models exhibited below. In the models, there is no uncertainty, there is a complete set of futures markets, and there are no externalities. But it is precisely uncertainty, the lack of futures markets and insurance markets, and externalities, which motivate influencing the investment vector of an economy away from its laissez-faire value. Our apology consists in conjecturing that the conclusions about planning that we arrive at here will be preserved in more realistic and complex models.

Incentive issues related to asymmetric information and the politics of a market socialist society are beyond the scope of the present chapter; for discussion of these issues and their relation to the models of this chapter, see Bardhan and Roemer (1992).

In organizing the chapter, we have followed the practice of Sen (1971). Sections 2 through 6 present the analysis informally and verbally; the asterisked sections with the same numbers which follow present the formal definitions and theorems. Proofs are available separately (Ortuño-Ortin, Roemer and Silvestre, 1991).

2 The economic environment

There are $N + 1$ sectors in the economy and two time periods. There is one firm in each sector. Firm 0 produces the only investment good in the economy, and operates only in the first period. Firm j ($j = 1, \ldots, N$) produces a consumption good in each period. In the first period, each of the $N + 1$ firms has a production function with only labor as an input. We view the capital stock as already fixed, and unchangeable in the first period. During the second period, the production for each consumer-good firm is a function of investment and labor. These firms place orders for the investment good in period one, and make their investments in period two.

There are M citizens, each with a utility function defined over the N consumption goods in each period: there are $2N$ arguments in each utility function. In particular, we assume for simplicity that labor (or leisure) is not an argument in the utility function: each citizen offers the entire endowment of his labor in each period, so long as the wage is positive. There is only one kind of labor in the model; thus, differential skill is only imperfectly modeled as differential labor endowment. Each citizen has an entitlement to a share of profits of each firm, his "social dividend." In principle, there can be $M(N + 1)$ such shares. In a socialist economy, the government may opt to set simply a share of *total* profits going to each citizen, and so there would be only M such shares delineated: we call this the *simple socialist case*. The setting of shares, in any case, is the outcome of a political process. (Lange suggested that a household's social dividend be proportional to its size. We shall comment on this below.) In the *general case*, which includes that of a capitalist economy with government intervention (a social democracy), the shares of profits going to citizens vary across firms as well.

The timing of economic activity is as follows. Wages are paid at the end of each production period, and commodities are sold at the end of the period in which they are produced. Profits are paid to citizens at the end of the period in which they arise. Consumer-good firms order the investment good during the first period, but investment is not accounted as a cost of production until the second period, for it enters into production only then. Since firms must distribute (or turn over to the state) profits at the end of the first period, they must borrow to finance investment, because the investment good must be purchased at the end of the first period. Citizens, on the other hand, consume in each period, and also save in the first period; an equilibrium condition in the economy will be that total savings of citizens equals total investment of firms. At the end of the second period, firms pay back principal and interest, and citizens count the matured loans with interest as income in the second period.

Total labor in the first period must be allocated between the production of the investment good and consumer goods. The larger the fraction of its labor an economy allocates to investment, the smaller its consumption will be in the first period, and the larger it will be in the second period, since investment augments the production of output in the second period. A government which seeks to raise

levels of investment, in this model, is thus concerned to replace first-period consumption with second-period consumption.

3 Constrained Walrasian equilibrium: a command-market-thought experiment

Suppose the government, which wishes to implement a given vector of investments across sectors, were able costlessly to monitor firms. A natural way of accomplishing its goal would be as follows: to command each consumer-good firm to invest exactly the desired amount, and to allow markets to do the rest. We call the market equilibrium associated with this procedure an *exactly-constrained Walrasian (ECW) equilibrium*. It is, formally speaking, a set of prices and an allocation which constitute a Walrasian equilibrium, with one restriction: that each consumer-good firm has no choice but to invest in the way commanded. In particular, it might well be at such an equilibrium that some firms register negative profits, for they are being commanded to invest more than they would if they were unrestricted. Thus, firms do not have the option of shutting down, for shutting down entails investing a zero amount. When profits are negative for a firm at an ECW equilibrium, the losses must be paid for by citizens, according to their assigned shares of firm profits. In this case, the entitlement to a firm's profits becomes an obligation to cover the firm's losses.

To be slightly more precise, an *exactly-constrained Walrasian* (ECW) *equilibrium relative to a vector of sectoral investments* $\bar{I} = (\bar{I}_1, \ldots, \bar{I}_N)$ is a set of prices, an interest rate and an allocation such that:

(1) Each consumer-good firm j demands labor and supplies output to maximize the present value of profits over two periods at given prices, subject to its demanding the investment good exactly in amount $\bar{I}_j$; the investment-good firm simply demands labor and supplies the investment good to maximize profits at given prices (in particular, it receives no command from the government).
(2) Each citizen receives labor income and his share of (constrained) profits of each firm, and chooses consumption and savings to maximize his utility over both periods subject to his budget constraints (one for each period).
(3) All markets clear. (In particular, the supply of the investment good by firm 0 equals the total investment demanded by the consumer-good firms, and total citizens' savings equals the total cost of the investment good.)

If the government chooses to implement an investment vector which is very large compared to the unconstrained Walrasian investment allocation for the economy, there may well exist no ECW equilibrium relative to that investment vector. The reason: investment might be so high that, at any candidate vector of prices, the total profits of consumer-good firms are negative, and indeed so negative that some citizens face a negative net income (their wage income does not suffice to pay their share of corporate losses). Thus, there will exist some set of investment vectors that can be implemented as ECW equilibria for a given economic environment, and in general this set will be strictly smaller than the set of *feasible* investment vectors for the economic environment. A feasible investment vector is one which the economy is technologically capable of realizing. Indeed, we view the maximal feasible total level of investment as the amount of investment good that can be produced with the economy's entire supply of labor in the first period. (This assumes that citizens can survive with no consumption in the first period; otherwise, we must alter the maximum feasible amount of investment appropriately.) The maximal feasible level of investment can be divided among consumer-good firms arbitrarily (again, assuming there are no restrictions on the pattern of feasible consumption in the second period).

A second thought experiment relaxes the requirement that the firm invest *exactly* the amount commanded by the government, and replaces it with the requirement that the firm invest *at least* the amount commanded. Thus, each firm may elect to invest autonomously above the commanded amount, if that would increase profits at the given prices. We analogously define a *constrained Walrasian* (CW) *equilibrium relative to* $\bar{I}$ as a set of prices and an allocation which satisfy (1)–(3) above, but with the relaxed restriction. There will be a set of investment vectors that can be implemented as constrained Walrasian equilibria. We are here interested in the aggregate level of investment in each sector (commanded plus autonomous investment). Note that every constrained Walrasian allocation is also an exactly-constrained Walrasian allocation. For if an aggregate investment vector $\bar{I}$ arises as a CW equilibrium, it also can be viewed as an ECW equilibrium, at the same prices, where the government commands firms to invest exactly at the levels of $\bar{I}$. The set of constrained Walrasian allocations is thus a proper sub-set of the set of exactly-constrained Walrasian allocations. The exactly-constrained Walrasian concept is interesting only if the government wants to limit investment in some firms to levels that are lower than their Walrasian levels, roughly speaking: for if it is concerned only to increase investment levels, it might as well set lower bounds only on the amount a firm must invest.

Although the set of investment vectors that can be implemented as constrained (or exactly-constrained) Walrasian equilibria is not as large as the set that could be implemented by a Stalinist system in which labor is directly allocated to the investment good industry, we take it to be a natural set of investment vectors to aim at implementing with market socialist techniques: for it relies upon the market to decentralize all resource allocation – not only the allocation and pattern of final consumption, but the allocation of labor to firms. In particular, as a consequence of the use of markets we have: any constrained (resp. exactly-constrained) Walrasian equilibrium relative to $\bar{I}$ is Pareto-efficient among the set of feasible allocations in which sectoral investments are greater than or equal to (resp. precisely equal to) $\bar{I}$.[5]

The command mechanism used here, however, is highly unrealistic, as in reality firms cannot be costlessly monitored, and it is hard to imagine that management of firms could be easily motivated to engage in production plans which entail losses. In particular, we shall assume throughout our discussion that

incentives have been created that induce managers to maximize profits – such incentives would work counter to the implementation of CW (or ECW) allocations, and so some other mechanism must be used if we are to implement the CW allocations realistically.

In the next two sections, we propose alternative mechanisms which indeed can implement the set of allocations (and investment vectors) that can be theoretically reached as CW and ECW equilibria, but in ways that firms achieve the desired result by maximizing profits subject to no commands from the center.

4 A generalization of Lange's idea

Once it has a viable market, the state should have reliable leverage to influence economic processes. This includes, in the first place, a rational profit tax system for enterprises and an income tax system for the general population, financial controls, the state bank's regulation of money turnover as a single whole, and an active credit policy, including lending rates corresponding to actual economic conditions. (Mikhail S. Gorbachev[6])

Lange wrote that the state bank should generate the desired investment level in the economy by setting the interest rate at which firms can borrow. Since our planner desires to implement a vector of investment levels, her instrument will be a vector of N discounts (or surcharges) on the market interest rate: one at which each consumer-good firm can borrow to finance investment.

Suppose that the planner wishes to increase all investments of consumer-good firms above their unconstrained Walrasian levels. Let the Walrasian interest rate be r. Assuming that all other prices remain fixed at their Walrasian levels, the planner can increase the investments chosen by firms by posting interest rate discounts for them. This will increase the demand for investment. To generate the required savings from citizens to finance the increased demand for investment funds, the market interest rate must rise above r. But then the central bank will suffer a deficit at the end of the second period, for it collects interest at low rates from firms at that time, and must pay out interest at the higher market rate to citizens. The deficit must be covered by taxing citizens' incomes. We allow the planner to tax only corporation profits to cover the deficit.

With this motivation, we define a *Lange equilibrium with corporate taxation relative to a given vector of sectoral investments* $\bar{I}$ as a vector of prices and wages, a vector of N interest rate discounts, and a vector of tax rates[7] on the profits of the N consumer-good firms such that the following hold:

(1) Each consumer-good firm maximizes the present value of profits. It discounts its second-period profits by the interest rate appropriate for citizens (the "market rate"), since profit income is distributed to them; but it debits the cost of investment at the corporate interest rate it is charged. The investment-good firm simply maximizes period-one profits. All profits are sent to the center.
(2) The vector of investment levels (autonomously) chosen by firms is $\bar{I}$.
(3) The center disburses profits to citizens, according to their entitlements, after

levying taxation at the stated rates for each sector. If a citizen's net after-tax social dividend is negative, then he receives a tax bill from the government, instead of a positive dividend.

(4) All consumers choose consumption in each period and savings in the first period to maximize utility over both periods, subject to their budget constraints, which include wage income, social dividends, and interest from savings.

(5) All markets clear.

Figure 17.2 presents the pattern of money flows in the economy.

In a Lange equilibrium, the subsidy to firm j is the cost of its expenditure on investment times the interest discount that the firm enjoys: for this is precisely the deficit that the central bank sustains from financing the firm's investment expenditure with high-interest loans from citizens. Note that the tax levied on a firm's profits under this scheme may be greater than its profits. In this case, of course, the (negative) after-tax profits are charged to citizens according to their shares. Or, if the planner wishes to discourage investment by a particular firm, that firm will face an interest rate surcharge.

We define a Lange equilibrium as *pro-investment* if all firms receive (nonnegative) interest rate discounts (not surcharges). Our first theorem states: (A) in the simple socialist case, the set of CW allocations is exactly the set of pro-investment Lange equilibria; and (B) in the simple socialist case, the set of ECW allocations is exactly the set of Lange equilibria. Thus, in the simple socialist case the Lange model provides a *decentralized, non-command mechanism by which everything can be implemented that can be implemented using the command structure of the previous section.*[8] In particular, all firm managers are instructed, in the Lange model, to maximize profits facing prices and effective interest rates.[9]

In the general case (i.e., when the profit shares differ across firms as well as across citizens), the Lange model is *more powerful* than the exactly-constrained Walrasian model: the set of allocations that can be implemented with the former strictly includes the set that can be implemented with the latter.

We envisage the mechanism working as follows. The center announces the N interest rate discounts from the market rate for the firms in the various sectors, and the after-tax net dividends to consumers; all prices adjust until markets clear. Unlike Lange, we provide no tâtonnement story for convergence to equilibrium. Our view that the market "finds" the Lange equilibrium is as justified as the standard view that the unregulated market finds the Walrasian equilibrium.

It is worthwhile to examine the special case of Lange equilibrium in which the corporate profits' tax rates are constrained to be less than one. This, in particular, will guarantee that citizens always receive nonnegative social dividends. We call such an equilibrium a *limited-taxation Lange equilibrium relative to $\bar{I}$* to distinguish it from the general case when corporate tax rates can be greater than one. In the simple socialist case we have (Theorem 4.2): the set of allocations (and investment vectors) that can be implemented as limited-taxation Lange equilib-

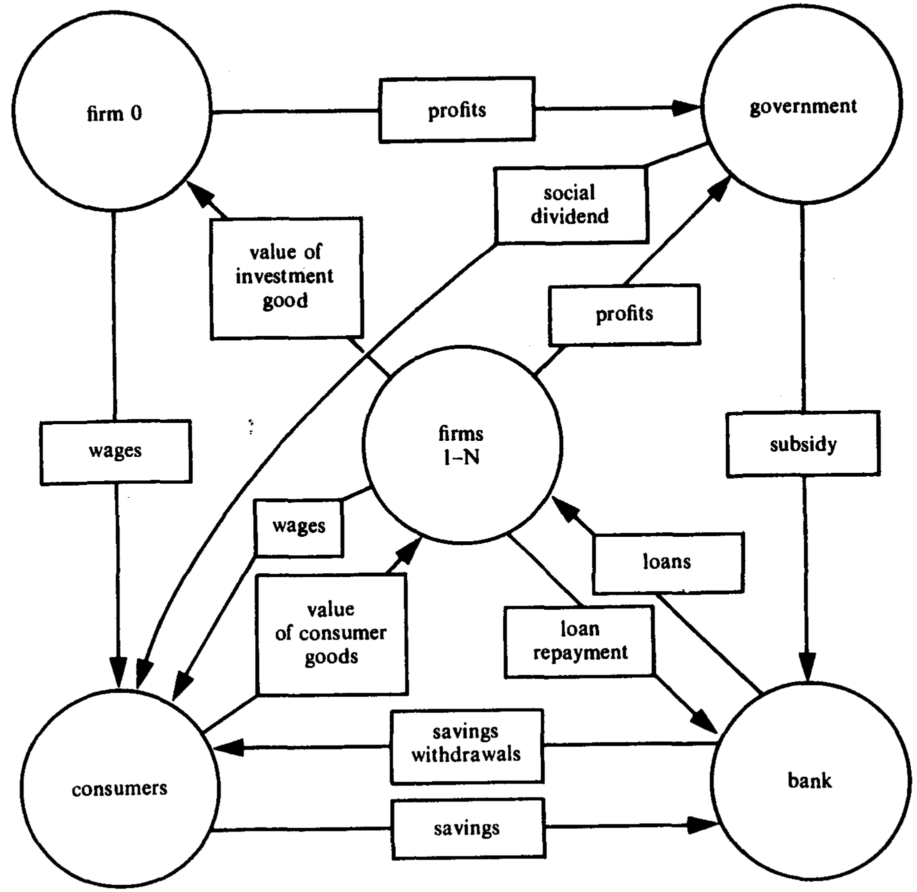

Figure 17.2 Money flows in the Lange model

ria is precisely the set of exactly-constrained Walrasian allocations in which total consumer-good firms' profits are nonnegative; and the set of allocations that can be implemented as limited-taxation pro-investment Lange equilibria is precisely the set of constrained Walrasian allocations in which total consumer-good firms' profits are nonnegative.

In any market socialist model, there will be opportunities for arbitrage, or black markets, for the government is interfering with the market mechanism. In the Lange model, the black market would be a capital market among consumer-good firms: firms that can borrow at a low interest rate could profit by lending to firms that can borrow only at a higher rate. This must be monitored by the center. There may also be opportunities for arbitrage between citizens who want to borrow (rather than save) but can do so only at the market rate, and firms that can borrow (and therefore lend) at low rates. In particular, firms might be tempted to lend to their workers under mutually advantageous arrangements.

We initially suggested that the shares of total profits distributed to households be viewed as a supplement to wage income. We have now seen, however, that if the government wishes to implement an investment vector at which total profits of firms are less than the total subsidy, then the aforementioned shares become citizens' obligations to finance corporate subsidies from their wage income. In particular, a vector of large sectoral investments may be implementable only by assigning large shares of corporate profits (i.e., shares of after-tax corporate debt) to citizens with large labor endowments. More generally, the point is that a political party's proposal for a schedule of shares may not be independent of the investment plan it proposes to implement.

5 Direct provision of investment by the state

We next take what seems to be a more direct approach to implementing the center's investment targets. In the Lange model, the center acts only by interfering with the market interest rate. In the model to be described now, it acts as a large economic agent, purchasing the investment good on the market and distributing it, *gratis*, to consumer-good firms. It finances the purchase of the investment good by levying taxes on corporate profits. Firms are free to purchase more of the investment good if they must to maximize profits; they know the amount of investment they will receive from the state. This scheme is reminiscent of Brus' (1972, Ch. 5) proposal of "a planned economy with a built-in market mechanism."

To be precise, a *direct investment equilibrium relative to* $\bar{I}$ is a set of prices, wages, an interest rate, a set of corporate tax rates, and an allocation at which:

(1) Every firm maximizes profits, choosing labor demands, outputs, and autonomous investment levels, knowing that it will receive the prescribed amount of investment, $\bar{I}_j$, from the state.
(2) All profits are distributed, net of corporation taxes, to citizens. Each citizen maximizes utility over two periods, subject to his budget constraints.
(3) Total taxes collected exactly finance the investment good purchased by the state.
(4) All markets clear.

We again specialize the direct investment concept to *limited-taxation direct investment equilibrium*, in which we restrict corporate tax rates to be between zero and one.

The first theorem is quite analogous to our result in section 4: in the simple socialist case, the set of direct-investment allocations is equal to the set of constrained Walrasian allocations, and the set of limited-taxation direct investment allocations is equal to the set of constrained Walrasian allocations *in* which total profits of consumer-good firms are nonnegative.

We conclude our analysis of the Lange and direct investment models by comparing the set of allocations that can be implemented with each of them. Theorem 5.2 states that: (A) in general, the set of allocations that can be implemented as limited-taxation pro-investment Lange equilibria is a proper sub-set of the set of limited-taxation direct investment equilibria. Thus, the direct investment mechanism is in principle more powerful than the Lange mechanism, as long as the center wants to encourage (rather than discourage) investment.[10] (If it wants to discourage some investment levels, then it will set an interest rate surcharge for some firms; the direct investment model is useless if discouragement is desired.) (B) However, the increased power of the direct investment mechanism *vis-à-vis* the Lange mechanism vanishes *in the simple socialist case*, for in that case we have that *the sets of limited-taxation Lange allocations and limited-taxation direct investment allocations are identical.*

We do not conclude that, in practice, these two implementation mechanisms – Lange and direct investment – would be equivalent in the simple socialist case. For they may well differ in certain aspects that we have ignored at the level of abstraction of our analysis. For instance, we have assumed that the center has available all the data describing the economic environment, and hence can calculate exactly the values of the instruments, such as interest rates, required to implement the desired investment vector. In reality, the center has only econometric and survey data. Suppose it announces the "wrong" vector of interest rate discounts, in the Lange model. It watches the markets, and adjusts the discounts as it sees what actually happens. This is relatively easy, and we can imagine that the economy will end up at some Lange equilibrium reasonably close to the one the center was aiming at, assuming its original information on demand functions and technologies was reasonably good. But in the direct investment model, the center commits itself to make certain purchases of the investment good, and to certain deliveries to firms. It may be much harder to adjust these obligations.

We may summarize the practical conclusion of sections 3, 4, and 5 as follows: in the simple socialist case, any allocation that can be implemented by the interventionist techniques of the command-market or direct investment mechanisms can also be implemented by the informationally superior, less interventionist Lange mechanism.

6 The sales-tax model

As in the Lange model, the government objective is to implement exactly a given strictly positive vector $\bar{I} = (\bar{I}_1, \ldots, \bar{I}_N)$ of sectoral investment. Again as in the Lange model, the firms follow price signals. But now the government instruments are taxes on sales, i.e., the prices faced by the firm differ from those faced by consumers. Because the present instrument requires the government to collect taxes (from citizens) instead of giving subsidies (to firms), running the scheme now generates a surplus instead of a deficit. Balancing the budget now requires the transfer of funds to the citizens, instead of from them. Balancing the budget does not therefore conflict with the need to keep the wealth of citizens positive.

The previous existence difficulties associated with this conflict thus do not appear here. Actually, existence of equilibrium can now be proved given an arbitrary rule for transferring the government surplus to citizens.

Another difference from the Lange model is that now all firms face the same input prices. This implies productive efficiency: contrary to the Lange and direct investment models, in a sales-tax equilibrium it is not possible to increase the output of all consumer goods by a mere reallocation of inputs.

We view the government as choosing $2N$ *ad valorem* tax rates, one for each consumer good in each period. This introduces a discrepancy between the price received by the producer and the price paid by the consumer (the latter being equal to the former multiplied by one plus the tax rate). Thus, the description of an equilibrium may specify the $2N$ producer prices and either the $2N$ tax rates or, equivalently, the $2N$ prices faced by consumers. We choose the second convention. The complete description of a *sales-tax equilibrium relative to the vector of sectoral investments $\bar{I}$ and to a given rule of (lump-sum) transfer of the government surplus* is given by (a) wage rates for the first and second periods, (b) a price for the investment good, interpreted as both the price received by the firm that produces it and the price paid by the firms that use it as an input, (c) first- and second-period producer prices for the N goods sold to consumers, (d) first- and second-period consumer prices for the same goods, and (e) an allocation such that:

(1) the profit-maximizing supply of the investment good (given the investment-good price and the wage) equals the sum of the components of $\bar{I}$
(2) the profit-maximizing demand for the investment good by each consumer-good firm (given the wage rates, the price of the investment good and the two-period prices of the consumer good faced by the firm) equals the relevant component of the vector $\bar{I}$
(3) the wealth of each citizen, which she takes as given, equals the value of her labor endowments, plus her profit income (evaluated at the prices faced by firms), plus her share in the government surplus as determined by the given transfer rule
(4) the two-period consumption of each citizen maximizes her utility subject to the constraint that the value of her two-period consumption, evaluated at consumer prices, does not exceed her wealth.

We adopt the following strategy for tackling the existence issue (Theorem 6.1). Set the first-period wage equal to one (i.e., first-period labor is used as numéraire), and set the price of the investment good at the level that induces its producer to supply exactly the required amount (namely, the sum of the components in the vector $\bar{I}$). The choice of first-period supplies of consumer goods is basically unrestricted: any positive amounts that use up the labor resource left after producing the investment good will do. After choosing these supplies, set the first-period producer prices at the level that induces the supply of precisely these quantities. Now we impose an assumption guaranteeing that the marginal rate of input substitution goes to zero with the amount of labor input used: a fixed-point argument guarantees that, under this assumption, an allocation of the second-period labor resource exists that equalizes the marginal rates of input substitution across firms when each firm is using the amount of investment good in $\bar{I}$. Because the price of the investment good is already given, this common marginal rate yields the appropriate second-period wage. By setting output prices equal to marginal costs we complete the description of the quantities supplied and producer prices. We are left with the description of consumer prices and quantities.

To this end, choose an arbitrary number α not lower than the value of the two-period output of consumer goods evaluated at the just specified producer prices. A second fixed-point argument shows the existence of a two-period vector of consumer prices that (a) clears the markets and (b) puts the aggregate value of the two-period output of consumer goods at precisely α dollars.

Intuitively, the center now has $2N$ instruments, namely, the tax rates on each consumption good in periods one and two, whereas it has only N objectives, the investment levels in N sectors. This leaves N degrees of freedom. The center may, in particular, induce any allocation among the N sectors using the amount of first-period labor left after producing the prescribed quantity of the investment good, using up $N - 1$ degrees of freedom. Moreover, it can, without affecting the amounts of each good produced, arbitrarily choose its total tax receipts. This suggests that a rich variety of allocations can be obtained as equilibria of the sales-tax model, for a given investment vector $\bar{I}$. There may, in particular, be large government surpluses which empower the given transfer rule with a significant distributional role. The sales-tax mechanism becomes particularly flexible if the transfer rule is not *a priori* given but can be chosen as a policy instrument.

Finally, we address the comparison of the direct investment and the sales-tax schemes. The comparison is not obvious because direct investment equilibria are second-best efficient under the sectoral investment constraints, but they typically display productive inefficiency (shifting both capital and labor among sectors could increase production). Sales-tax equilibria display productive efficiency and they distribute the produced amounts efficiently, but they are not second-best efficient relative to the sectoral investment constraints (shifting labor among sectors could yield a different output mix that could be distributed so as to Pareto-dominate the original allocation). But it turns out that the sales-tax mechanism is superior to the direct investment one in the following sense (Theorem 6.2): given a direct investment equilibrium, one can find a transfer rule with a sales-tax equilibrium that yields the same aggregate level of investment (i.e., the same sum of the components in $\bar{I}$) yet (weakly) Pareto-dominates the original direct investment equilibrium.

A sales-tax scheme may present serious political problems, especially when the mechanism involves large discrepancies between producer and consumer prices (i.e., large government surpluses). We conclude with a quotation from Lange (1938:96–97) on these difficulties:

There would be two sets of prices of consumers' goods. One would be the market prices at which the goods are sold to the consumers; the other, the accounting prices derived from the preference scale fixed by the Central Planning Board. The latter set of prices would be those on the basis of which the managers of production would make their decisions.

However, it does not seem very probable that such a system would be tolerated by the citizens of a socialist community. The dual system of prices of consumers' goods would reveal to the people that the bureaucrats in the Central Planning Board allocate the community's productive resources according to a preference scale different from that of the citizens.

We concur with this view, and believe that the direct investment or Lange mechanism may be politically more acceptable, at least domestically, in particular when financed by a corporation tax. It must be mentioned, however, that the use of interest rate or direct investment subsidies to firms producing internationally traded goods will draw accusations of unfair practices from governments of rival producers.

7 Conclusion

We have offered some preliminary, if lengthy, explorations on methods by which the center may implement a given target of sectoral investments in an economy where (a) firms maximize profits, (b) goods and labor are allocated through markets, and (c) profit shares are exogenously given. Two types of society are covered: first, a social democratic one, where firms are privately owned, profit shares simply reflect private property rights, and the state has an activist investment policy; second, a market socialist economy, where firms are publicly owned, managers are instructed to maximize profits, and profit shares are parameters, generated by a democratic process, for the distribution of the social surplus. We define the *simple socialist case* to be a socialist economy where profit shares do not vary across firms.

Our object is to study the range of investment vectors or allocations that can be achieved through the following instruments: (i) central commands to firms, (ii) interest rates faced by firms, as suggested by *Lange*; (iii) *direct investment* by the center; and (iv) *sales-tax* rates. The center must always balance its budget. We endow the center with the capability to tax profits as it sees fit for the amount needed to cover the cost of its investment program.

We consider command mechanisms, which share the feature of leaving to the market the allocation of labor and consumer goods while the center issues commands on investment. In the first one, called the *constrained Walrasian* mechanism, the center imposes a lower bound on the investment of each industry, but firms are free to invest more if they so wish. Thus, the mechanism is unable to force low levels of investment. In the second mechanism, called *exactly-constrained Walrasian*, the center commands exactly the investment level of the firm, and, hence, it can force both high and low investment levels. The *Lange* mechanism can in principle set interest rates for firms that are lower or higher than the market rate, and, as an effect, it may induce both high and low investment levels. We define the *pro-investment Lange* mechanism by requiring that the interest rates faced by firms do not exceed the market rate.

A first result is that, in the general case, *the Lange mechanism is more powerful than the exactly-constrained Walrasian mechanism.* (In a parallel manner, the pro-investment Lange mechanism is more powerful than the constrained Walrasian mechanism.) Thus, the indirect instruments of credit subsidy and profit taxation give the center more flexibility than direct commands on investment. The intuition is that the command method affects incomes only via investment, whereas the Lange mechanism has two tools: credit conditions, which affect both investment levels and incomes, and taxes, which are an independent instrument for income redistribution. This extra flexibility is, however, lost *in the simple socialist case*, where *the Lange mechanism is equivalent to the exactly-constrained one*. This is so because, in the simple socialist case, varying tax rates across firms has no effect on the tax bill of a consumer.

One should moreover note that these results are based on a strong form of taxation, because the center is able to tax profits at a rate exceeding 100 percent, in effect also taxing wage income. If this strong form of taxation is ruled out, and if attention is restricted to the *simple socialist case*, then the *Lange mechanism can achieve the same allocations that the exactly-constrained Walrasian mechanism can achieve with nonnegative aggregate profits* (and, in a parallel manner, the pro-investment Lange mechanism can achieve the same allocations that the constrained Walrasian can with nonnegative profits).

Next, we consider the direct investment mechanism. The center now distributes the investment good free of charge to firms, and does not restrict the autonomous investment of firms: it can induce high, but not low, investment levels. In particular, it may be unable to implement some low-investment states that could be reached either by exact constraints or by the Lange mechanism (via high effective interest rates). What about its power to encourage, rather than discourage, investment? The relevant comparison is between the pro-investment Lange mechanism and the direct investment mechanism, but with taxation limited to rates not exceeding 100 percent. This comparison gives sharp results: *the two mechanisms are equivalent in the simple socialist case, while in the general case the direct investment mechanism is, in principle, more powerful.*

Lastly, we consider the sales-tax mechanism, where the center may impose a divergence between the prices paid by consumers and those received by firms. It turns out that *the sales-tax mechanism can implement a large variety of allocations and investment vectors*, but it may require large price differences that are politically problematic. Thus, this scheme cannot easily be compared with the others.

While the direct investment mechanism often emerges here as not inferior to the Lange one, and sometimes superior to it, our abstractions may conceal some reasons favoring the interest rate-guided Lange approach. Two specific extensions of the analysis deserve study. First, the path-breaking work by Weitzman (1974) suggests that the superiority of price or quantity instruments under limited information may well depend on the curvature of the relevant functions. Second, if there are many firms in each industry, then sectoral investment targets may perhaps be implemented more simply via interest rates than by individual quantity signals. We conclude with an invitation to further research.

2* The economic environment

There are $N + 1$ firms and two periods. Firm 0 produces an investment good in the first period. Firm j, for $j = 1, \ldots, N$, produces a consumption good in each period. In the first period, the production function of firm j is $f_j^1(L)$, for $j = 0, \ldots, N$, where L is labor; in the second period, the production functions are $f_j^2(I, L), j = 1, \ldots, N$ where I is the investment good. For $j = 0, \ldots, N$, we assume that the function f_j^1 is concave, strictly increasing, and differentiable on R_{++}, and satisfies $f_j^1(0) = 0$. For $j = 1 \ldots, N$, we assume that f_j^2 is concave, strictly increasing, and differentiable on R^2_{++}, and satisfies $f_j^2(0,0) = 0$.

There are M consumers. The utility function of consumer i is $u_i(x_1^1, \ldots, x_N^1, x_1^2, \ldots, x_N^2)$ where x_j^t is the consumption of good j in period t; the endowment of consumer i is a vector of labor inputs in the two periods, $(\bar{L}_i^1, \bar{L}_i^2)$ and a vector $(\theta_{i0}, \ldots, \theta_{iN})$ where θ_{ij} is i's share of firm j's profits. If our model is describing a social democracy,[11] then θ_{ij} has the usual interpretation as an ownership share. If we are describing a socialist economy, then we interpret the profits that i receives as a social dividend, to use Lange's phrase, in which case the vectors $\{\theta_i | i = 1, \ldots, M\}$ are chosen by the center. (According to Lange, θ_{ij} should be chosen to reflect need, perhaps in proportion to family size.) Note that consumers derive no utility from leisure; thus, consumer i always supplies his entire labor endowment.

We denote a *feasible allocation* as a tuple $(x^1, x^2, y^1, y^2, L^{1D}, L^{2D}, I)$ where $x^t \in R_+^{NM}$ for $t = 1, 2$, $y^1 = (y_0^1, y_1^1, \ldots, y_N^1) \in R_+^{N+1}$, $y^2 = (y_1^2, \ldots, y_N^2) \in R_+^N$, $L^{1D} = (L_0^{1D}, \ldots, L_N^{1D}) \in R_+^{N+1}$, $L^{2D} = (L_1^{2D}, \ldots, L_N^{2D}) \in R_+^N$, $I = (I_1, \ldots, I_N) \in R_+^N$, satisfying

$$\sum_{i=1}^{M} x_{ij}^t \leqq y_j^t, \quad j = 1, \ldots, N; t = 1, 2,$$

$$\sum_{j=1}^{N} I_j \leqq y_0^1,$$

$$f_j^1(L_j^{1D}) = y_j^1, \quad j = 0, \ldots, N,$$

$$f_j^2(I_j, L_j^{2D}) = y_j^2, \quad j = 1, \ldots, N,$$

$$\sum_{j=0}^{N} L_j^{1D} \leqq \sum_{i=1}^{M} \bar{L}_i^1, \quad \text{and}$$

$$\sum_{j=1}^{N} L_j^{2D} \leqq \sum_{i=1}^{M} \bar{L}_i^2.$$

Thus, x_{ij}^t is the consumption of good j by consumer i in period t; y_j^t is the production of firm j in period t; L_j^{tD} is the demand for labor by firm j in period t, etc. We will often denote vectors by deletion of a subscript: thus $\theta_i = (\theta_{i0}, \ldots, \theta_{iN})$, $\bar{L}^1 = (\bar{L}_1^1, \ldots, \bar{L}_M^1)$, $x_i^1 = (x_{i1}^1, \ldots, x_{iN}^1)$, etc. Similarly, prices are denoted (p^1, p^2, w^1, w^2) where $p^1 = (p_0^1, p_1^1, \ldots, p_N^1) \in R_+^{N+1}$, $p^2 = (p_1^2, \ldots, p_N^2) \in R_+^N$, are goods' prices and $w^t \in R_+$, $t = 1, 2$, are wages. We write $p_{-0}^1 = (p_1^1, \ldots, p_N^1)$, and $p_{-0} = (p_{-0}^1, p^2)$.

We will almost always take the Arrow–Debreu viewpoint and treat all trading decisions as made at the beginning of period 1, facing a vector of futures prices (p^1, p^2, w^1, w^2). Thus each consumer has one budget constraint over his "lifetime" and each firm maximizes the sum of profits over the two periods. The amounts that consumers save and firms borrow are therefore not explicitly shown, and the interest rate is hidden in the price vector.

3* Constrained Walrasian equilibrium: a command-market-thought experiment

Let $\bar{I} = (\bar{I}_1, \ldots, \bar{I}_N)$ be a given investment vector. Imagine the center commands firm j to invest precisely $\bar{I}_j$ $(j = 1, \ldots, N)$; at an *exactly-constrained Walrasian* (ECW) *equilibrium relative to* $\bar{I}$, all markets clear, all consumers maximize utility, and all firms maximize profits subject to their investment constraints. Formally:

Definition 3.1
The price vector (p^1, p^2, w^1, w^2) and the allocation $(\bar{x}^1, \bar{x}^2, \bar{y}^1, \bar{y}^2, \bar{L}^{1D}, \bar{L}^{2D}, \bar{I})$ constitute an *ECW equilibrium relative to* $\bar{I}$ if:

(1) $(\bar{y}_0, \bar{L}^{1D})$, solves firm O's profit-maximization program:

$$\max \bar{p}_0^1 y_0^1 - \bar{w}^1 L_0^{1D}$$
$$\text{s.t.} \quad f_0^1(L_0^{1D}) \geqslant y_0^1;$$

$(\bar{y}_j^1, \bar{y}_j^2, \bar{L}_j^{1D}, \bar{L}_j^{2D})$ solves firms j's constrained profit-maximization program:

$$\max \bar{p}_j^1 f_j^1(L_j^{1D}) - \bar{w}^1 L_j^{1D} + \bar{p}_j^2 f_j^2(\bar{I}_j, L_j^{2D}) - \bar{w}^2 L_j^{2D} - \bar{p}_0^1 \bar{I}_j;$$

(2) $(\bar{x}_i^1, \bar{x}_i^2)$ solves consumer i's utility-maximization program:

$$\max u_i(x_i^1, x_i^2)$$
$$\text{s.t.} \quad \bar{p}_{-0}^1 \cdot x_i^1 + \bar{p}^2 \cdot x_i^2 \leqslant \bar{w}^1 \bar{L}_i^1 + \bar{w}^2 \bar{L}_i^2 + \sum_{j=0}^{N} \theta_{ij} \pi_j,$$

where

$$\pi_0 = \bar{p}_0^1 \sum_{j=1}^{N} \bar{I}_j - \bar{w}^1 \bar{L}_0^{1D} \text{ and, for } j = 1, \ldots, N,$$

$$\pi_j = \sum_{t=1}^{2} (\bar{p}_j^t \bar{y}_j^t - \bar{w}^t \bar{L}_j^{tD}) - \bar{p}_0^1 \bar{I}_j;$$

(3) All markets clear; for example the market for investment goods clears,

$$\sum_{j=1}^{N} \bar{I}_j = \bar{y}_0^1.$$

In particular, consumers' wealths must be nonnegative at an ECW equilibrium, a substantive requirement, since it is possible that total profits are negative for some price vectors.

That an ECW equilibrium is Pareto-optimal subject to the constraint that the investment vector be precisely $(\bar{I}_1,\ldots,\bar{I}_N)$ follows from the usual proof of the "first welfare theorem."

Next, we define a constrained Walrasian (CW) equilibrium relative to $\bar{I}$ as a Walrasian equilibrium in which firm j ($j = 1,\ldots,N$) is required to invest at least $\bar{I}_j$. Formally:

Definition 3.2
A *CW equilibrium relative to* $\bar{I}$ is a price vector $(\bar{p}^1,\bar{p}^2,\bar{w}^1,\bar{w}^2)$ and allocation $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\bar{L}^{1D},\bar{L}^{2D},\bar{R})$ such that

(1) Firm 0 maximizes profits at $(\bar{y}_0^1,\bar{L}_0^{1D})$; for $j = 1,\ldots,N$, $(\bar{L}_j^{1D},\bar{L}_j^{2D},\bar{R}_j)$ solves firm j's profit-maximization program:

$$\max \bar{p}_j^1 f_j^1(L_j^{1D}) - \bar{w}^1 L_j^{1D} + \bar{p}_j^2 f_j^2(R_j,L_j^{2D}) - \bar{w}^2 L_j^{2D} - \bar{p}_0^1 R_j$$
$$\text{s.t.}\quad R_j \geqq \bar{I}_j;$$

(2) All consumers maximize utility;
(3) All markets clear.

As above, the virtue of CW equilibria is that they are second-best Pareto-optimal, i.e., Pareto-optimal subject to the constraint that the vector of investments $\bar{R}$ is greater than or equal to $\bar{I}$.

4* A generalization of Lange's idea

Our planner desires to implement a vector $\bar{I}$ of investment levels; her instrument will be a vector $(d_1,\ldots,d_N)\in R_+^N$, where d_j is the interest rate discount that firm j receives from the market rate; the financing scheme will be a vector $(\tau_1,\ldots,\tau_N)\in R_+^N$ of corporate tax rates. Recall from section 2 that firms must borrow to finance investment.

We write down the definition of Lange equilibrium using futures prices, where the interest rate discounts are not immediately evident:

Definition 4.1
A *Lange equilibrium relative to* $\bar{I} = (\bar{I}_1,\ldots,\bar{I}_N)$ is a vector of prices $(\bar{p}^1, \bar{p}^2,\bar{w}^1,\bar{w}^2)$, a vector $\lambda = (\lambda_1,\ldots,\lambda_N)$ in R_+^N, a vector of corporation profits' tax rates $(\tau_1,\ldots,\tau_N)\in R_+^N$ and an allocation $(\bar{x}^1,\bar{x}^2,\bar{y}^1,\bar{y}^2,\bar{L}^{1D},\bar{L}^{2D},\bar{I})$ such that:[12]

F1 $(\bar{x}_i^1,\bar{x}_i^2)$ solves:

$$\max u_i(x_i^1,x_i^2)$$
$$\text{s.t.}\quad \bar{p}_{-0}^1\cdot x_i^1 + \bar{p}^2\cdot x_i^2 \leqslant \bar{w}^1\bar{L}_i^1 + \bar{w}^2\bar{L}_i^2 + \theta_{i0}\bar{\pi}_0 + \sum_{j=1}^{N}(1-\tau_j)\theta_{ij}\bar{\pi}_j$$

where $\bar{\pi}_j$ is the value of firm j's profits, given in F2a and F2b below:

F2a $(\bar{L}_0^{1D},\bar{y}_0^1)$ solves:

$$\max \bar{p}_0^1 y_0 - \bar{w}^1 L_0^{1D}$$
$$\text{s.t.}\quad f_0(L_0^{1D}) \geqq y_0^1;$$

F2b For $j = 1,\ldots,N$, $(\bar{L}_j^{1D},\bar{y}_j^1,\bar{L}_j^{2D},\bar{y}_j^2,\bar{I}_j)$ solves:

$$\max \bar{p}_j^1 y_j^1 - \bar{w}^1 L_j^{1D} + \bar{p}_j^2 y_j^2 - \bar{w}^2 L_j^{2D} - \lambda_j\bar{p}_0^1 I_j$$
$$\text{s.t.}\quad f_j^1(L_j^{1D}) \geqslant y_j^1,$$
$$f_j^2(I_j,L_j^{2D}) \geqslant y_j^2;$$

F3 $$\sum_{j=1}^{N}\tau_j\bar{\pi}_j = \sum_{j=1}^{N}(1-\lambda_j)\bar{p}_0^1\bar{I}_j;$$

F4 $$\bar{y}_0^1 = \sum_{j=1}^{N}\bar{I}_j,$$
$$\bar{y}_j^t = \sum_{i=1}^{M}\bar{x}_{ij}^t,\quad j = 1,\ldots,N,\ t = 1,2,$$
$$\sum_{j=0}^{N}\bar{L}_j^{tD} = \sum_i \bar{L}_i^t,\quad t = 1,2.$$

Interpretation
$\lambda_j = \dfrac{1+r_j}{1+r_C}$, where r_j is the interest rate charged for loans to firm j, and r_C is the (market) interest rate faced by consumers. Thus λ_j acts as a discount on the price of the investment good (see F2b above), if the government subsidizes an industry, or a surcharge, if it wishes to depress investment in an industry. The expression on the r.h.s. of F3 is the total subsidy; F3 states that corporation taxes exactly cover the total subsidy.

Definition 4.2
A *pro-investment Lange equilibrium relative to the investment vector* $\bar{I}$ is a Lange equilibrium in which, for $j = 1,\ldots,N$, $\lambda_j \leqslant 1$.

Define $\mathscr{A}^L$ to be the set of Lange equilibria relative to some investment vector, and $\mathscr{A}^{L[0,1]}$ to be the set of pro-investment Lange equilibria.

Theorem 4.1[13]

A. *(Simple socialist case) If* $\theta_{ij} = \theta_{ij'}$, *for all* $i = 1,\ldots,M$ *and for all* $j, j' = 1,\ldots,N$, *then*

$$\mathscr{A}^L = \mathscr{A}^{ECW} \quad \textit{and} \quad \mathscr{A}^{L[0,1]} = \mathscr{A}^{CW}.$$

B. *(General case)* $\mathscr{A}^L \supset \mathscr{A}^{ECW}$, $\mathscr{A}^{L[0,1]} \supset \mathscr{A}^{CW}$, *and the inclusions are strict.*

We next introduce a second specialization of the definition of Lange equilibrium.

Definition 4.3
A *limited-taxation Lange equilibrium relative to* $\bar{I} = (\bar{I}_1, \ldots, \bar{I}_N)$ is a Lange equilibrium such that:

$$0 \leqq \tau_j \leqq 1, \quad \text{for all } j = 1, \ldots, N.$$

Similarly, a limited-taxation pro-investment Lange equilibrium is a limited-taxation equilibrium where for all j, $\lambda_j \leqq 1$.

Let $\mathscr{A}^{LTL}$ be the set of allocations that arise as limited-taxation Lange equilibria, and $\mathscr{A}^{LTL[0,1]}$ be the allocations that arise as limited-taxation pro-investment Lange equilibria. Let $\mathscr{A}^{CW+}$ be the set of constrained Walrasian allocations where total profits in the consumer-good industries are nonnegative.

Theorem 4.2 (*Simple socialist case*)
If $\theta_{ij} = \theta_{ij'}$, *for all* $i = 1, \ldots, M$ *and for all* $j, j' = 1, \ldots, N$, *then*

$$\mathscr{A}^{LTL[0,1]} = \mathscr{A}^{CW+},$$

$$\mathscr{A}^{LTL} = \mathscr{A}^{ECW+}.$$

5* Direct provision of investment by the state

Definition 5.1
A direct investment (DI) *equilibrium with corporate taxation relative to* $\bar{I}$ consists of prices $(\bar{p}^1, \bar{p}^2, \bar{w}^1, \bar{w}^2)$ an allocation $(\bar{x}^1, \bar{x}^2, \bar{y}^1, \bar{y}^2, \bar{L}^{1D}, \bar{L}^{2D}, \bar{M} + \bar{I})$, and tax rates $\tau = (\tau_1, \ldots, \tau_N) \in R^N_+$ on profits such that:

D1 $(\bar{y}_0, \bar{L}_0^{1D})$ solves firm 0's profit-maximization program:

$$\max \bar{p}_0^1 y_0^1 - \bar{w}^1 L_0^{1D}$$
$$\text{s.t. } f_0(L_0^1) \geqslant y_0^{1D};$$

$(\bar{y}_j^1, \bar{y}_j^2, \bar{L}_j^{1D}, \bar{L}_j^{2D})\bar{M}_j$ solves, for $j = 1, \ldots, N$,

$$\max \bar{p}_j^1 y_j^1 + \bar{p}_j^2 y_j^2 - \bar{w}^1 L_j^{1D} - \bar{w}^2 L_j^{2D} - \bar{p}_0 M_j$$
$$\text{s.t. } f_j^1(L_j^{1D}) \geqslant y_j^1,$$
$$f_j^2(\bar{I}_j + M_j, L_j^{2D}) \geqslant y_j^2,$$
$$M_j \geqslant 0;$$

D2 $(\bar{x}_i^1, \bar{x}_i^2)$ solves consumer i's utility-maximization program:

$$\max u_i(x_i^1 x_i^2)$$
$$\text{s.t. } \bar{p}^1_{-0} \cdot x_i^1 + \bar{p}^2 \cdot x_i^2 \leqslant \bar{w}^1 \bar{L}_i^1 + \bar{w}^2 \bar{L}_i^2 + \theta_{i0}\pi_0 + \sum_{j=1}^{N} \theta_{ij}(1 - \tau_j)\pi_j,$$

where $\pi_j = \bar{p}_j^1 \bar{y}_j^1 + \bar{p}_j^2 \bar{y}_j^2 - \bar{w}^1 \bar{L}_j^{1D} - \bar{w}^2 \bar{L}_j^{2D} - \bar{p}_0^1 \bar{M}_j$ for $j = 1, \ldots, N$ and $\pi_0 = \bar{p}_0^1 \bar{y}_0^1 - \bar{w}^1 \bar{L}_0^{1D}$;

D3 $\sum_{j=1}^{N} \tau_j \pi_j = \bar{p}_0^1 \sum_{j=1}^{N} \bar{I}_j$ (the government's budget constraint);

D4 All markets clear; for instance, the market for investment goods clears,

$$\sum_{j=1}^{N} (\bar{M}_j + \bar{I}_j) = \bar{y}_0^1.$$

Note that all firms make nonnegative profits at a DI equilibrium.

As in section 4*, we define the following specialization of the above definition.

Definition 5.2
A *limited-taxation direct investment equilibrium relative to* $\bar{I}$ is a direct investment equilibrium such that, for all $j = 1, \ldots, N$, $0 \leqslant \tau_j \leqslant 1$.

Let $\mathscr{A}^{DI}$ be the set of direct investment allocations with corporate taxation relative to some $\bar{I}$. Let $\mathscr{A}^{LTDI}$ be the set of limited taxation DI-equilibrium allocations with corporate taxes.

Theorem 5.1 (*Simple socialist case*)
If for all $i = 1, \ldots, M$ *and* $j, j' = 1, \ldots, N$, $\theta_{ij} = \theta_{ij'}$ *then*

A. $\mathscr{A}^{CW} = \mathscr{A}^{DI}$.
B. $\mathscr{A}^{CW+} = \mathscr{A}^{LTDI}$.

Theorem 5.2

A. $\mathscr{A}^{LTL[0,1]} \subset \mathscr{A}^{LTDI}$. *In general the inclusion is strict.*[14]
B. *The simple socialist case, if for all* $i = 1, \ldots, M$ *and* $j, j' = 1, \ldots, N$, $\theta_{ij} = \theta_{ij'}$ then $\mathscr{A}^{LTL[0,1]} = \mathscr{A}^{LTDI} = \mathscr{A}^{CW+}$.

6* The sales-tax model

Assumption 6.1
The receipts T from taxes are returned to the consumers according to a given transfer rule $S: R \to R^M$ satisfying:

(a) for all $T \in R$, $\sum_{i=1}^{M} S_i(T) = T$;
(b) if $T \geqslant 0$, then $S_i(T) \geqslant 0$, $i = 1, \ldots, M$.

Remark 6.1
Assumption 6.1 is satisfied by the *proportional transfer rule* $S_i(T) = \sigma_i T$, for $(\sigma_1, \ldots, \sigma_M) \in R^M_+$ satisfying $\sum_{i=1}^{M} \sigma_i = 1$.

Remark 6.2
We interpret both T and $S_i(T)$ as measured in nominal terms, i.e., relative to an arbitrary unit of account. This allows the subsidy rule to be numéraire-dependent, although the proportional subsidy rate is not. Alternatively, one may avoid this feature by interpreting that both T and $S_i(T)$ are measured in real terms, i.e., in the units of a good selected as numéraire.

Remark 6.3
Sales receipts T could in principle be negative, i.e., the consumer purchases of some goods are subsidized by taxes on other goods and lump-sum taxes on consumers. Theorem 6.1 below will actually show the existence of an equilibrium with nonnegative T.

$$\text{Define:}\quad MRS_j(L_j) = \frac{(\partial f_j^2/\partial I_j)|(\bar{I}_j, L_j)}{(\partial f_j^2/\partial L_j^{2D})|(\bar{I}_j, L_j)}.$$

Assumption 6.2
For $j = 1,\ldots,N$, and $\bar{I}_j > 0$, $\lim_{\varepsilon\to 0} MRS_j(\varepsilon) = 0$.

Assumption 6.3
For $i = 1,\ldots,M$, $\bar{L}_i^1 > 0$, and u_i is strictly quasi-concave and strictly increasing.

Definition 6.1
Let $\bar{I} \gg 0$. A *second-period production equilibrium relative to* $\bar{I}$ is a vector $(\bar{w}^1, \bar{w}^2, \bar{p}_0^1, \bar{p}^2, \bar{L}^{2D}) \in R_+ \times R_+ \times R_+ \times R_+^N \times R_+^N$ satisfying

(i) $\sum_{j=1}^N \bar{I}_j$ solves $\max_y \bar{p}_0^1 y - \bar{w}^1 (f_0^1)^{-1}(y)$;

(ii) for $j = 1,\ldots,N$, $(\bar{I}_j, \bar{L}_j^{2D})$ solves $\max_{I_j, L_j} \bar{p}_j^2 f_j^2(I_j, L_j) - \bar{p}_0^1 I_j - \bar{w}^2 L_j^{2D}$;

(iii) $\sum_{j=1}^N \bar{L}_j^{2D} = \sum_{i=1}^M \bar{L}_i^2$.

Remark 6.4
If $(\bar{w}^1, \bar{w}^2, \bar{p}_0^1, \bar{p}^2, \bar{L}^{2D})$ is a second-period production equilibrium, then so is $(\lambda\bar{w}^1, \lambda\bar{w}^2, \lambda\bar{p}_0^1, \lambda\bar{p}^2, \bar{L}^{2D})$ for any $\lambda > 0$.

Lemma 6.1
Under Assumption 6.2, if $\bar{I} \gg 0$ *and* $\sum_{j=1}^N \bar{I}_j < f_0^1\left(\sum_{i=1}^M \bar{L}_i^1\right)$, *then a second-period production equilibrium relative to* $\bar{I}$ *exists (with* $\bar{w}^1 = 1$*).*

Proof (sketch)
Define, for $j = 1,\ldots,N$,

$$\phi_j : R_+ \to R_+ : \phi_j(L_j) = \begin{cases} 0 & \text{if } L_j = 0 \\ MRS_j(L_j) & \text{if } L_j > 0. \end{cases}$$

and

$$\beta_j(L) = (1/N)\sum_{i=1}^N \phi_i(L_i) - \phi_j(L_j),\quad j = 1,\ldots,N;$$

Write $\hat{L} = \sum_{i=1}^M \bar{L}_i^2$. Define the $(N-1)$ dimensional simplex $\Delta_L = \left\{L \in R_+^N \,\middle|\, \sum_{j=1}^N L_j = \hat{L}\right\}$, and the continuous mapping $\Lambda : \Delta_L \to \Delta_L$,

$$\Lambda_j(L) = \frac{L_j + \hat{L}\max\{0, \beta_j(L)\}}{1 + \sum_{i=1}^N \max\{0, \beta_i(L)\}}.$$

A fixed point of Λ yields the desired second-period allocation of labor, see Ortuño-Ortin, Roemer and Silvestre (1991) for details.

Remark 6.5
Assumption 6.2 is indispensable for the validity of Lemma 6.1, see Example 3 in Ortuño-Ortin, Roemer and Silvestre (1991).

We view the government as choosing $2N$ tax rates $(\tau_1^1,\ldots,\tau_N^1,\tau_1^2,\ldots,\tau_N^2)$, where τ_j^t is the tax on good j at period t. Producer prices $p_{-0} = (p_1^1,\ldots,p_N^1,p_1^2,\ldots,p_N^2)$ are then related to consumer prices $\psi = (\psi_1^1,\ldots,\psi_N^1,\psi_1^2,\ldots,\psi_N^2)$ by the *sales-tax equalities*:

$$\psi_j^t = (1+\tau_j^t)p_j^t,$$

or:

$$\tau_j^t = \frac{\psi_j^t - p_j^t}{p_j^t},$$

where τ_j^t can in principle range from -1 to $+\infty$.

Consumer i takes as given his wealth W_i and the consumer prices ψ. He chooses $x_i = (x_i^1, x_i^2) \in R_+^{2N}$ in order to maximize u_i on $\gamma(\psi, W_i) = \{x_i \in R_+^{2N} | \psi\cdot x_i \leqslant W_i\}$. This set is closed and convex, and it is nonempty for $W_i \geqslant 0$, but it fails to be compact when $\psi_j^t = 0$ for some t,j. Define:

$$k = 2\sum_{j=1}^N \left[f_j^1\left(\sum_{i=1}^M \bar{L}_i^1\right) + f_j^2\left(f_0^1\left(\sum_{i=1}^M \bar{L}_i^1\right), \sum_{i=1}^M \bar{L}_i^2\right)\right],$$

$$K = \{(x_i^1, x_i^2) \in R_+^{2N} | x_{ij}^t \leqslant k \text{ for } j = 1,\ldots,N \text{ and } t = 1,2\},$$

and:

$$\bar{\gamma}(\psi, W_i) = \gamma(\psi, W_i) \cap K.$$

Define the functions:

$$\xi_i : R_{++}^{2N} \times R_+ \to R^{2N} : \xi_i(\psi, W_i) = \text{“the unique maximizer of } u_i \text{ on } \gamma(\psi, W_i)\text{,”}$$

and:

$$\xi_i : R_+^{2N} \times R_+ \to R^{2N} : \xi_i(\psi, W_i) = \text{"the unique maximizer of } u_i \text{ on } \gamma(\psi, W_i)\text{."}$$

These functions are well defined, because they involve maximizing a continuous function on a compact set and because the uniqueness of the maximizer is guaranteed by strict quasi-concavity and strict monotonicity.

Write $\bar p_{-0} = (\bar p_1^1, \dots, \bar p_N^1, \bar p_1^2, \dots, \bar p_N^2)$ and $\bar y_{-0} = (\bar y_1^1, \dots, \bar y_N^1, \bar y_1^2, \dots, \bar y_N^2)$.

Definition 6.2

A sales-tax equilibrium relative to a (strictly positive) *vector of sectoral investments* $\bar I = (\bar I_1, \dots, \bar I_n)$ *and to a transfer rule* $\bar S$ is a vector $(\bar p^1, \bar p^2, \bar\psi^1, \bar\psi^2, \bar w^1, \bar w^2, \bar x^1, \bar x^2, \bar y^1, \bar y^2, \bar L^{1D}, \bar L^{2D}) = (\bar p, \bar\psi, \bar w, \bar x, \bar y, \bar L) \in R^{2N+1} \times R^{2N} \times R^2 \times R^{2MN} \times R^{2N+1} \times R^{2N+1}$ such that:

(i) $(\bar y_0^1, \bar L_0^{1D})$ maximizes $\bar p_0^1 y - \bar w^1 L$ subject to $y = f_0^1(L)$;

(ii) for $j = 1, \dots, N$, $(\bar I_j, \bar L_j^{1D}, \bar L_j^{2D})$ solves:

$$\max_{(I_j, L_j^{1D}, L_j^{2D})} \quad \bar p_j^1 f_j^1(L_j^{1D}) - \bar w^1 L_j^{1D} + \bar p_j^2 f_j^2(I_j, L_j^{2D}) - \bar w^2 L_j^{2D} - \bar p_0^1 I_j;$$

(iii) for $j = 1, \dots, N$, $\bar y_j^1 = f_j^1(\bar L_j^{1D})$ and $\bar y_j^2 = f_j^2(\bar I_j, \bar L_j^{2D})$;

(iv) $\bar y_0^1 = \sum_{j=1}^{N} \bar I_j$;

(v) $\bar\psi \gg 0$, and, for $i = 1, \dots, M$, $\bar x_i = \xi_i(\bar\psi, \bar W_i)$, where

$$\begin{aligned} \bar W_i = {} & \bar w^1 \bar L_i^1 + \bar w^2 \bar L_i^2 + \theta_{i0}(\bar p_0^1 \bar y_0^1 - \bar w^1 \bar L_0^1) \\ & + \sum_{j=1}^{N} \theta_{ij}(\bar p_j^1 \bar y_j^1 + \bar p_j^2 \bar y_j^2 - \bar w^1 \bar L_j^{1D} - \bar w^2 \bar L_j^{2D} - \bar p_0^1 \bar I_j) \\ & + S_i((\bar\psi - \bar p_{-0}) \cdot \bar y_{-0}); \end{aligned}$$

(vi) $\sum_{j=0}^{N} \bar L_j^{1D} = \sum_{i=1}^{M} \bar L_i^1$;

$$\sum_{j=1}^{N} \bar L_j^{2D} = \sum_{i=1}^{M} \bar L_i^2;$$

(vii) $\sum_{i=1}^{M} \bar x_{ij}^t = \bar y_j^t; \quad j = 1, \dots, N, \ t = 1, 2.$

Remark 6.6

Let S be proportional (see Remark 6.1 above), and assume that $(\bar p, \bar\psi, \bar w, \bar x, \bar y, \bar L)$ is an equilibrium relative to $\bar I$ and that $\lambda \gg 0$. Then $(\lambda\bar p, \lambda\bar\psi, \lambda\bar w, \bar x, \bar y, \bar L)$ is an equilibrium for $\bar I$. Of course, the same tax rates $\tau_j^t = (\bar\psi_j^t - \bar p_j^t)/\bar p_j^t$ apply to both equilibria. This is no longer true when S is numéraire-dependent, see Remark 6.2 above.

We now tackle the existence issue. Choose a positive second-period production equilibrium $(\bar w^1, \bar w^2, \bar p_0^1, \bar p^2, \bar L^{2D})$, which will be kept fixed in what follows (Lemma 6.1 guarantees that this is possible). Write $\bar y_0^1 = \sum_{j=1}^{N} \bar I_j$ and $\bar L_0^{1D} = (f_0^1)^{-1}(\bar y_0^1)$. The choice of first-period magnitudes is basically unrestricted: let $(\bar L_1^{1D}, \dots, \bar L_N^{1D})$ be *any* strictly positive vector satisfying

$$\sum_{j=1}^{N} \bar L_j^{1D} = \sum_{i=1}^{M} \bar L_i^1 - \bar L_0^{1D}.$$

For $j = 1, \dots, N$, write $\bar y_j^1 = f_j^1(\bar L_j^{1D})$ and choose $\bar p_j^1$ so that $\bar y_j^1$ solves: $\max_y \bar p_j^1 y - (f_j^1)^{-1}(y)$. This choice is again possible by concavity. For future reference, we call a vector $(\bar L_1^{1D}, \dots, \bar L_N^{1D}, \bar p_1^1, \dots, \bar p_N^1)$ a *first-period production equilibrium* (we take $\bar w^1$ to be one). These choices of first- and second-period production equilibria define a vector $(\bar p, \bar w, \bar y, \bar L)$, to be called a *production equilibrium* (relative to $\bar I$), which will be kept fixed in what follows. For $i = 1, \dots, M$, define:

$$\begin{aligned} \tilde W_i(\psi) = {} & \bar w^1 \bar L_i^1 + \bar w^2 \bar L_i^2 + \theta_{i0}(\bar p_0^1 \bar y_0^1 - \bar w^1 \bar L_0^{1D}) \\ & + \sum_{j=1}^{N} \theta_{ij}(\bar p_j^1 \bar y_j^1 + \bar p_j^2 \bar y_j^2 - \bar w^1 \bar L_j^{1D} - \bar w^2 \bar L_j^{2D} - \bar p_0^1 \bar I_j) \\ & + S_i((\psi - \bar p_{-0}) \cdot \bar y_{-0}), \end{aligned}$$

$$\tilde\xi_i(\psi) = \xi_i(\psi, \tilde W_i(\psi)) \in R^{2N},$$

and

$$\zeta(\psi) = \sum_{i=1}^{M} \tilde\xi_i(\psi) - \bar y_{-0} \in R^{2N}.$$

Lemma 6.2 (*Walras' Law*)

Under Assumptions 6.1 and 6.3, for all $\psi = (\psi^1, \psi^2) \in R^{2N}$, $\psi \cdot \zeta(\psi) = 0$.

Theorem 6.1

Under Assumptions 6.1–6.3, an equilibrium relative to $\bar I$ ($\bar I \gg 0$) *and S exists.*

Proof (*sketch*)

Take an arbitrary positive number α satisfying: $\alpha \geqq \bar p_{-0} \cdot \bar y_{-0}$, and define the simplex $\Delta_\alpha = \{\psi \in R_+^{2N} \mid \psi \cdot \bar y_{-0} = \alpha\}$, which is homeomorphic to the standard $2N - 1$ dimensional simplex because $\bar y_j^t > 0$ ($j = 1, \dots, N$, $t = 1, 2$). Define the continuous function: $\Psi : \Delta_\alpha \to \Delta_\alpha$,

$$\Psi_j^t(\psi) = \frac{\psi_j^t + (\alpha / \bar y_j^t) \max\{0, \zeta_j^t(\psi)\}}{1 + \sum_{i=1}^{N} \sum_{v=1}^{2} \max\{0, \zeta_i^v(\psi)\}}.$$

A fixed point of Ψ yields the desired consumer prices. See Ortuño-Ortin, Roemer and Silvestre (1991) for details.

The proof of Theorem 6.1 shows that for a $(\bar w, \bar p, \bar y, \bar L^D)$ satisfying the conditions of a production equilibria in both periods one can obtain a (at least one-dimensional) continuum of equilibria by varying the parameter α in the proof as long as $\alpha \geqq \bar{p}_{-0} \cdot \bar{y}_{-0}$. Our proof restricts α to be not lower than $\bar{p}_{-0} \cdot \bar{y}_{-0}$, thus ensuring that tax receipts are nonnegative and, hence, that the wealth of consumers is positive. If $\alpha < \bar{p}_{-0} \cdot \bar{y}_{-0}$, then an equilibrium with $\bar{y}_{-0} \cdot \psi = \alpha$ may or may not exist. It will not exist if the data of the economy are such that the wealth of a consumer becomes negative for negative and large (in absolute value) tax receipts.

Now we show that, given a direct investment equilibrium with aggregate level of investment K^* (i.e., $K^* = \Sigma \bar{I}_i + \Sigma \bar{M}_i$), there is a sales-tax equilibrium that yields the same aggregate investment K^* yet (weakly) Pareto-dominates the original direct investment equilibrium.

Theorem 6.2
Let $\bar{e} = (\bar{w}^1, 1, \bar{w}^2, \bar{p}^1, \bar{p}^2, \bar{x}^1, \bar{x}^2, \bar{y}^1, \bar{y}^2, \bar{L}^{1D}, \bar{L}^{2D}, \bar{I} + \bar{M})$ *be a direct investment equilibrium relative to* $\bar{I} \gg 0$. *Assume that the conditions of Theorem 6.1 are satisfied, and that*

$$\sum_{j=1}^{N} (\bar{I}_j + \bar{M}_j) < f_0^1(\bar{L}^1).$$

Then there exists a positive vector $\hat{I}$, *a proportional transfer rule* $\hat{S}$, $\hat{S}_i(T) = \hat{\sigma}_i T$, $i = 1, \ldots, M$, *and a sales-tax equilibrium* $\hat{e}$ *relative to* $\hat{I}$ *and* $\hat{S}$ *with associated consumption allocation* $\hat{x}$ *such that:*

(i) $\sum_{j=1}^{N} \hat{I}_j = \sum_{j=1}^{N} (\bar{I}_j + \bar{R}_j)$;

(ii) $u_i(\hat{x}_i^1, \hat{x}_i^2) \geqq u_i(\bar{x}_i^1, \bar{x}_i^2)$.

Notes

1. For a good précis of the history of the concept, see W. Brus (1987). For discussions of the reforms in the Soviet Union, see Hewett (1988) and Aslund (1989).
2. Negative externalities associated with pollution, for instance; positive externalities associated with "endogenous growth," as in Romer (1986).
3. Another possibility occurs to us to explain Lange's insistence that the state set prices of investment goods. It would have been too radical, within the socialist community, to propose that the state set only the interest rates, and leave everything else to the market. Socialist parties, whom Lange presumably hoped to influence, would not have been content with so apparently minor an interventionist role.
4. We assume throughout that the center knows all the parameters of the economy; we are not concerned here with the implementation problem which must take into account incentive compatibility.
5. Although constrained Walrasian allocations are second-best Pareto-optimal as described, they are not generally technologically efficient. The given composition of output is efficiently distributed among citizens; labor is efficiently allocated among firms, given their levels of investment; but it is generally possible to find a reallocation of investment among firms, and a reallocation of labor that would increase production of all outputs. Such reallocation of investment are, however, precluded given the government's investment objective.
6. In his speech after being sworn in to the Soviet presidency, as translated by Tass and reported in the *New York Times* of March 16, 1990.
7. Note that these tax rates may be greater than one.
8. Of course, Lange equilibria inherit the second-best Pareto-optimality of constrained Walrasian equilibria. Moreover, suppose the government wishes to implement only a given *aggregate* level of investment. It can accomplish this with just one interest rate discount for all firms. The resulting equilibrium will be technologically efficient, unlike constrained Walrasian equilibria. (It will still, however, be only second-best Pareto-optimal, as there will generally be reallocations of labor between the investment sector and the consumption sector that will produce a composition of output that can be distributed in a way to increase the utility of all citizens.) There is no natural way to implement a targetted aggregate level of investment in the command-market model of section 3. To whom would the center issue the command?
9. As we show in section 4*, the Lange model may also be interpreted atemporally, by using the device of futures' prices, in which case interest rates do not explicitly appear. In this version, it is as if the government refunds to the firm a portion of the cost of each unit of investment good the firm purchases. The price which the firm in sector j pays for the investment good is effectively discounted by the factor $\frac{1 + r_j}{1 + r_C}$, where r_j and r_C are the industry and market interest rates, respectively.
10. See, however, the caveat in n. 14 to Theorem 5.2 in section 5*.
11. We take a social democracy to be an economy with private ownership of firms in which the government is empowered to influence investment.
12. We assume that firm 0 is not taxed.
13. Proofs of all theorems are available in Ortuño-Ortin, Roemer and Silvestre (1991).
14. We show strict inclusion by providing an example where $N = 2$. Our example is not entirely satisfactory. It does, indeed, show the existence of an allocation that is implementable by direct investment but not by interest rates; however, the *investment vector* of the allocation is implementable in a Lange allocation. We have not found an example showing that the set of investment vectors implementable as Lange allocations is a proper sub-set of those implementable by direct investment. We have not searched in models with $N > 2$.